Chapter 2 Review of the Literature

There is little dispute that good subject knowledge is needed to teach mathematics effectively at secondary level, but the research into the importance of subject knowledge when teaching in primary schools is less clear in its findings (Ball 1990, Morris 2001, Goulding *et al.* 2003, Rowland and Turner 2008). Some of this lack of clarity lies in the complexity in defining subject knowledge and the difficulty in analysing the relative importance of subject knowledge in relation to attitudes, beliefs and pedagogical skills. It is also a matter of debate as to whether a primary teacher is able to have deep subject knowledge in the ten or more curriculum areas that they are expected to teach, raising the question of the place of the specialist teacher in primary schools (Alexander 2010).

How important is the basic subject knowledge and attitude to mathematics that primary teachers bring to the classrooms?

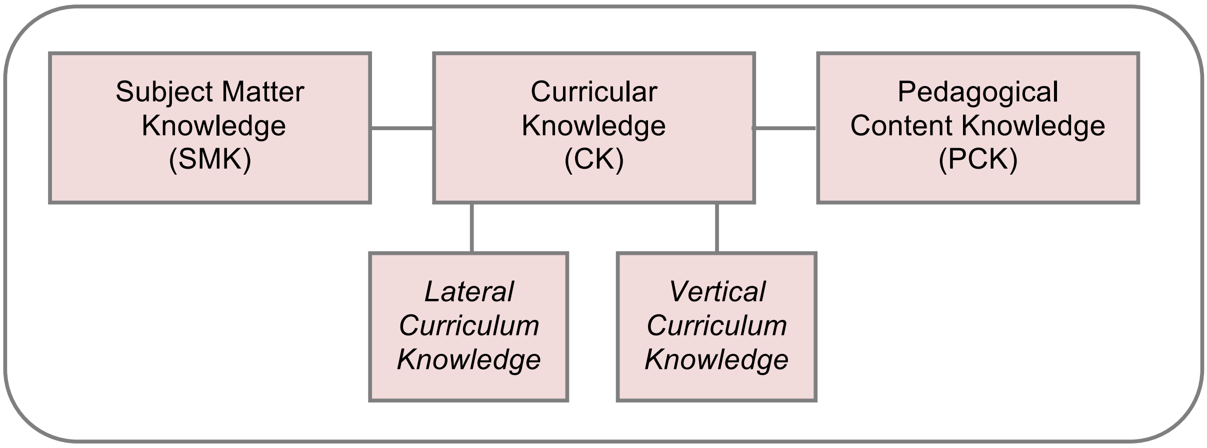
Clearly we all have a level of basic knowledge of mathematics through our own learning as pupils in school, which is taken on into adulthood. Most importantly we also develop beliefs about mathematics and attitudes towards the subject, and trainee teachers take these attitudes to college or university. Ball (1990) commented that, despite the intervention of teacher training institutes, teachers are most likely to teach mathematics just as they were taught themselves. Regardless of our attitudes towards mathematics there is a basic level of mathematics that teachers need in order to teach mathematics. Not surprisingly, teacher training courses in Higher Education Institutes have minimum mathematics entry requirements set nationally by the TDA, but concerns over the basic mathematical knowledge of trainee teachers was highlighted when the TDA made it a requirement for trainee teachers to pass a numeracy skills test to gain Qualified Teacher Status (TDA 2007). Subject knowledge booster courses were also provided, but it has been evident that many trainee primary teachers were worried about teaching mathematics, and in fact some had a fear of the subject from their own experiences in school (Aubrey 1994, Bibby 2002).

I share the view of Bibby that mathematics can actually have an emotional impact, with a palpable fear of failure and the shame felt at a lack of understanding of a particular aspect of mathematics. This was my own personal experience, and the attitude I took to college as a trainee teacher. The teacher training experience is an important step towards becoming an effective teacher, not only in gaining knowledge about the subject of mathematics, but also in developing confidence in mathematics and a positive attitude towards teaching it.

What subject knowledge is used to teach mathematics effectively?

Much of the research on teachers' subject knowledge follows on from the work of Lee Shulman (1986) in his studies of the development of trainee teachers in their final year at college through to becoming novice secondary teachers. He focussed on content knowledge and proposed a special domain of teacher knowledge that he termed *Pedagogical Content Knowledge (PCK)*. He suggested that there is content knowledge unique to teaching, which is subject-matter-specific and professional knowledge. Teachers need this knowledge to provide rich, purposeful, appropriate and rigorous mathematics for their pupils. To show the place of PCK, he identified three content-related categories, shown in Figure 2.1.

Figure 2.1: Shulman's categories of subject knowledge



The first, *Subject Matter Knowledge (SMK)*, includes the basic content knowledge of the subject as well as an understanding of the structure of that subject. It goes beyond knowledge of mathematical facts and concepts to include the reasons why a teacher makes judgements regarding, for example, the selection of a particular method to teach subtraction.

The second category, *Curricular Knowledge (CK)*, includes an understanding of the programmes of study and the resources available in relation to those programmes. Within this category there are two other dimensions of curricular knowledge that are important for teaching: *Lateral Curriculum Knowledge,* which is knowledge that relates the particular subject to other subject areas, and *Vertical Curriculum Knowledge* which relates to the knowledge of progression. Both of these touch upon the importance of making connections with other knowledge areas. This is a key aspect of effective teaching according to research at Kings College (Askew *et al.* 1997) and is also a feature of teachers' knowledge necessary for teaching (Rowland, Huckstep and Thwaites 2003).

The last of the three content-related categories is *Pedagogical Content Knowledge*. Shulman explains PCK as: '…the most useful ways of representing and formulating the subject that make it comprehensible to others' (Shulman 1986: 9). Teachers may have *good* subject knowledge but it is the effectiveness of the way they pass this on to their pupils that shows *deep* subject knowledge, or PCK if we see these as one and the same, through the use of representations, the quality of explanation and the types of questions. In whichever way that this knowledge is presented, it is certainly specialised mathematical knowledge for teaching.

Ball, Thames and Phelps (2008) built on Shulman's work with a focus on the mathematics subject knowledge of elementary teachers (Table 2.1). They concluded that it is essential that teachers know the subject they teach, but that this is very different from the mathematics that people other than teachers would recognise.

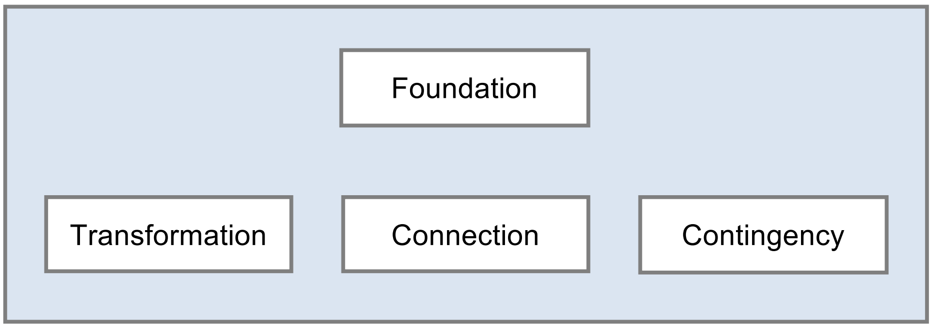
Table 2.1: Domains of mathematical knowledge for teaching

<table>
<tr><th colspan="2">SUBJECT MATTER KNOWLEDGE</th><th colspan="2">PEDAGOGICAL CONTENT KNOWLEDGE</th></tr>
<tr><td>Common content knowledge</td><td rowspan="2">Specialised content knowledge (SCK)</td><td>Knowledge of content and students</td><td rowspan="2">Knowledge of content and curriculum</td></tr>
<tr><td>Horizon content knowledge</td><td>Knowledge of content and teaching</td></tr>
</table>

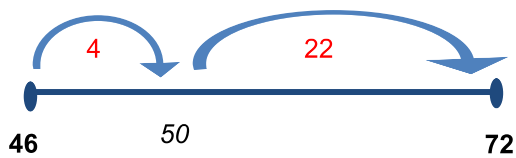
Horizon Content Knowledge relates to Shulman's lateral and vertical curriculum knowledge, recognising the importance of making connections with other areas of the curriculum and with different levels of mathematics. Ball *et al.* also introduced the domain of *Specialised Content Knowledge (SCK)* to describe the mathematical knowledge and skills unique to the teaching of mathematics. Although this does not make a very clear distinction from Shulman's PCK, an aspect of particular interest in this domain is the skill a teacher uses to 'unpack' the mathematics so that it makes sense to the learner. Teaching mathematics involves, amongst other things, an understanding of the conceptions and misconceptions that a pupil may encounter, and strategies to help counter these misconceptions. This is linked to an understanding of curriculum progression (Shulman's *Vertical Content Knowledge*) but puts a greater emphasis on the importance of the way the mathematics is represented or transformed to the learners, to secure understanding.

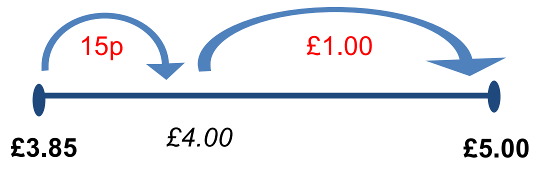
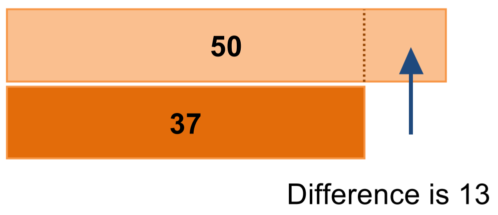
Rowland, Huckstep and Thwaites (2003) introduced *Transformation* as one of the four broad units through which mathematical knowledge of teachers could be observed, with a clear emphasis on the importance of representation (Figure 2.2).

Figure 2.2: Knowledge quartet



They identified a *Knowledge Quartet*, with *Foundation*, based on the mathematical knowledge the teacher already possesses and brings to the classroom, differing from the other three categories of *Transformation*, *Connection* and *Contingency* in that these three refer to the way knowledge is used in teaching.

Transformation is a stage within the process of planning and teaching, with the teacher's own knowledge, meanings and descriptions transformed and presented in ways designed to enable students to learn effectively. This includes the use of representation, explanation and demonstration. *Transformation* has a strong link with PCK and SCK, adding to these by making explicit the importance of teachers considering appropriate and specific methods, resources, language and questioning when introducing and teaching a topic.	*For example, when teaching subtraction to a class of nine- year-olds, a teacher may decide that counting on along a number line is the most effective way of representing 'finding the difference', which has been transformed from their foundation knowledge of using comparison to find differences.* *Through research and as part of the planning process they may decide to use blank number lines with different starting points and end points for children to count on to find differences.*  *The difference is 26*
Connection is an important element of teachers' deep subject knowledge. Within a single lesson, or across a series of lessons, the teacher draws out connections between different concepts or between alternative ways of representing concepts and carrying	*Using the 'counting on' example above, a teacher may make the connection between this and giving change with money (the shop-keepers' method) and use this to reinforce the number line method. So, to calculate the change from £5 of an item costing £3.85, the*

out procedures. This is similar to the domains of horizon content knowledge, lateral curriculum knowledge and vertical curriculum knowledge detailed earlier, but brings them together as a single key element.	*difference is calculated on a blank number line.*  *The change is £1.15*
Contingency is the ability of the teacher to 'think on one's feet' and respond appropriately to the contributions made by the pupils during a lesson. This was not made explicit in earlier research on subject knowledge (Shulman 1986, Ball 1990, Aubrey 1994), but it is an important aspect of teaching effectively; not only recognising and dealing with misconceptions, but also listening and responding to pupils' ideas. From this, teachers will perhaps gain deeper subject knowledge by understanding the knowledge that children bring to the lesson and to listen to them as they take an active part in their learning.	*Allowing discussion of the methods pupils use to solve a subtraction problem will give a teacher an opportunity to respond to any misconceptions that arise.* *For example, the term 'difference' may be confusing to a child with the number line image not supporting their understanding. A child may explain their thinking using the language of comparison, so a teacher could use a different model to help them understand.* 

From my own experience of classroom observations focussing on teachers' subject knowledge, the *Knowledge Quartet* is a useful model to use to support those observations. This was also a conclusion of the Cambridge Seminars (Ruthven and Rowland 2008).
As an Ofsted Inspector I was required to make judgements on the extent to which teachers showed good subject knowledge and understanding in the way they presented and talked about their subject (Ofsted 1999). This was always a challenging aspect of teaching and learning to evaluate, with the need to exemplify teachers' subject knowledge in relation to the impact on pupils' learning. The four units of *Foundation, Transformation, Connection and Contingency,* with their inherent characteristics, provide useful criteria to support the evaluation of a teachers' level of subject knowledge and I considered using them as a framework to support the research methods for this study. However, it is not necessarily a model designed to identify the characteristics of *deep* subject knowledge and was specifically aimed at providing a framework to use when observing teachers.

To help decide on an analytical framework for the observations and interviews I would be carrying out, I compared the different classifications of subject knowledge and then proposed my own simple taxonomy (Table 2.2).

Table 2.2: Comparison of subject knowledge frameworks

<table>
<tr><th>Shulman (1986)</th><th>Ball et al. (2008)</th><th>Aubrey (1994)</th><th>Rowland et al. (2003)</th><th>NCETM (2009)</th><th>My own taxonomy</th></tr>
<tr><td>Subject matter knowledge (SMK)</td><td>Common content knowledge</td><td></td><td>Foundation</td><td></td><td>Basic mathematical knowledge (BMK)</td></tr>
<tr><td>Curricular knowledge</td><td>Horizon content knowledge</td><td>Subject content knowledge</td><td rowspan="2">Connection

Transformation</td><td>Knowledge about mathematics</td><td rowspan="2">Knowledge of teaching mathematics (KTM)</td></tr>
<tr><td>Pedagogical content knowledge (PCK)</td><td>Specialised content knowledge (SCK)

Knowledge of content and teaching (KCT)</td><td>Pedagogical subject knowledge</td><td>Knowledge about ways of teaching mathematics</td></tr>
<tr><td>Knowledge of learners and their characteristics</td><td>Knowledge of content and students (KCS)</td><td>Knowledge of conceptions of pupils</td><td>Contingency</td><td>Knowledge about students' mathematical conceptions</td><td>Knowledge of learning mathematics (KLM)</td></tr>
</table>

Each row approximately corresponds to Shulman's original domains, and includes criteria on teachers' subject knowledge from the *Researching Effective CPD in Mathematics Education* (RECME) project (NCETM 2009). Within my own taxonomy, KTM and KLM are possible sub-sets of deep subject knowledge.

What is *deep* subject knowledge?

Williams (2008) highlighted the importance of deep subject knowledge, but the expression 'deep' has been acknowledged as a positive link with subject knowledge for a number of years. Ball (1990) acknowledged that, for effective teaching, teacher knowledge of mathematics needed to be *deep* and *flexible*, while Ma (1999) talks about the necessity for it to be *deep*, *vast* and *thorough*. But what does this depth of knowledge give to a teacher and how is it different to basic subject knowledge or, indeed, PCK?

This issue was discussed in part in a series of seminars in Cambridge on *Mathematical Knowledge in Teaching* (Ruthven and Rowland 2008), which aimed to draw together current ideas and evidence about mathematical knowledge. Much of the discussion related to initial and continuing professional development of teachers, endorsing the findings of the Williams review that professional development for teachers should focus on mathematical content, mathematical pedagogy and embedded practices (Williams 2008). Based on the discussions in some of these seminars and my experience supporting teachers as a curriculum advisor,
It is likely that the deep subject knowledge of an effective teacher of mathematics differs from basic content knowledge in the complex relationship that subject knowledge has with the pedagogical skills of that teacher. In fact it is very difficult to separate out subject knowledge used for teaching and the pedagogical skills that a teacher uses (McEwan and Bull 1991, McNamara 1991, Aubrey 1997b, Ruthven and Rowland 2008). Knowledge and pedagogy are distinct but overlapping aspects of teaching, with effective teachers building up and developing background knowledge about mathematics to inform their teaching in a broad way. This knowledge is then used to determine the most appropriate methods, representations, resources and activities (the pedagogical skills) to teach a specific aspect of that knowledge to the learner to implement the mathematics curriculum. Although knowledge and pedagogy are distinct, it is evident that they are very closely connected when considering the specific knowledge needed to teach mathematics effectively.

What is the connection between deep subject knowledge and pedagogy?

Teachers with deep subject knowledge are likely to have a good understanding of the progression within and connections between mathematical concepts, skills and facts, and the techniques, resources and strategies needed to teach these concepts, skills and facts effectively. This can be gained through observation and research as trainee teachers, but it is in the actual process of planning, teaching and reflecting on that teaching when deep subject knowledge and related pedagogical skills are likely to be developed. Williams (2008) focuses on promoting effective learning through the combination of deep subject knowledge and pedagogical skill. Supporting this, an inquiry into post-14 mathematics education (Smith 2004) emphasises the importance of broadening teachers' knowledge of subject-specific pedagogy through appreciating how pupils learn mathematics, recognising the role of questioning and an understanding of possible misconceptions. The report concludes that teachers should have the opportunity to reflect on the approaches they use to deliver the curriculum.

The connection between knowledge, pedagogy and the curriculum is at the heart of the current debate on the Primary Curriculum. A recent report from the Pearson Centre for Policy and Learning (McCulloch 2011) states that teachers need to be skilled in both subject knowledge and the processes that translate that knowledge into deep learning experiences. These are the pedagogical techniques and are essential in designing, planning and implementing the curriculum. This concurs with a study of international evidence for curriculum development and implementation which concludes that, for positive outcomes to the curriculum, professional development is necessary to support subject knowledge and the teaching and learning processes (Bell *et al.* 2008).

As Shulman (1989) recognised with the domain of PCK and through the research that followed, pedagogy evidently has a strong link with deep subject knowledge. Pedagogy is not simply 'teaching methods', as pointed out by Williams (2008), but it is the effective use of a repertoire of techniques, resources and strategies, along with a knowledge of mathematics teaching and beliefs held about the way it is taught and the way children learn mathematics. Alexander (2008, 2010) illustrates his concern about the lack of debate on pedagogy for teachers in the UK, by describing the contrasting emphasis on pedagogy and didactics in other countries. He suggests that there is an over-emphasis on the curriculum, which he turns around by including curriculum as 'what one needs to know' within his definition of pedagogy:

> '… the act of teaching together with its attendant discourse of educational theories, values, evidence and justifications. It is what one needs to know, and the skills one needs to command, in order to make and justify the many kinds of decision of which teaching is constituted.' (Alexander 2008: 47).

So how does deep subject knowledge impact on the decision-making of a teacher? Without deep subject knowledge it is possible that a teacher will lack confidence when teaching mathematics and 'play it safe', perhaps following unit plans or schemes without questioning the effectiveness of the planned activities or approaches for the pupils. A teacher would certainly find it more difficult or be less willing to deal with misconceptions and would be less able to apply curriculum knowledge appropriately. Opportunities may also be missed to allow pupils to explore and investigate from a rich mathematical starting point. At its worst, a teacher with poor subject knowledge may lack interest in the subject and convey feelings of negativity towards mathematics when teaching (Morris 2001, Bibby 2002, Swars *et al.* 2006).

How can teachers develop deep subject knowledge?

Much of the research on teachers' subject knowledge has a focus on Initial Teacher Training (ITT) and teachers just starting in the classroom (Ball 1990, Grossman 1990, Rowland, Huckstep and Thwaites 2003, Ellis 2007). The Education Secretary Michael Gove also focuses on ITT as an appropriate time to develop deep subject knowledge. One of the strategies for this is an introduction of skills tests before students are accepted on an ITT course from 2013 and, from September 2012, trainee teachers will have a higher pass mark with only three attempts to pass the tests (DfE 2011).

Williams (2008) proposes that deep subject knowledge mainly develops through the experience of teaching and through Continuing Professional Development (CPD). But what form should this CPD take? In an influential small-scale study, Aubrey (1994) states that teachers increase their mathematical knowledge through conducting their own enquiries about mathematics (syntactic) as distinct from increasing their knowledge of mathematics (substantive). This development of syntactic knowledge differs from the substantive knowledge of key facts, concepts and principles. Through enquiring, learning and reflecting on issues about mathematics, a teacher is likely to keep more informed on research and new developments in mathematics teaching and to critically evaluate any findings. Aubrey (1994) concludes that inadequate syntactic knowledge may lead to misrepresentation when teaching mathematics, leading to misconceptions for the learners. In a recent study on using research in the professional development of mathematics teachers, Els de Geest (2011) comes to a similar conclusion to Carol Aubrey. She suggests that comparing research findings within CPD gives teachers an awareness of different perspectives about teaching and learning. Using research helps to engage in deep thinking and to trust the validity of the ideas offered, encouraging the teachers to try the ideas in the classroom.

The National Centre for Excellence in the Teaching of Mathematics (NCETM) commissioned research into effective CPD in mathematics education (RECME Project, NCETM 2009). The report concludes that the use of research could play an important part in developing effective professional development, informing the programme and providing relevant material for the teachers to engage with. The report suggests that providers of CPD should include opportunities for teachers to develop knowledge about mathematics and ways of teaching, drawing upon relevant research. When identifying factors that contributed to effective professional development, the study found that some of the best teacher learning took place when the CPD dealt with all aspects of mathematical knowledge for teaching, including deep analysis of mathematics and detailed planning.

A further finding concerning subject knowledge and CPD from the RECME Project was that some teachers were reluctant to engage in any mathematics that was different to the levels they were teaching to their pupils, or that was not directly relevant to their current teaching. This is an interesting point and is related to the understanding and knowledge a teacher has about the connected nature of mathematics. Teachers who have poor basic subject knowledge are less likely to understand or utilize the connections in mathematics (Askew *et al.* 1997) and will also feel less comfortable working on mathematics away from their current teaching experience. Anne Watson held a different view when she delivered one of the Cambridge Seminars on mathematical knowledge in teaching (Watson 2008). She stated that CPD should concern the learning of mathematics, not purely on mathematics for teaching, with the experience of learning mathematics at a personal level an effective way to deepen and develop mathematical knowledge for teaching.

Does a high level of qualification in mathematics provide a primary teacher with deep subject knowledge?

Research on the impact of higher mathematics qualifications on a teachers' subject knowledge has largely been restricted to trainee teachers. Rowland *et al.* (2009) found that trainee teachers with a good knowledge of mathematics through higher qualifications were more competent mathematics teachers on their placements. Conversely, those with poor mathematics subject knowledge performed less competently in the classrooms. Aubrey (1997b) states that a fundamental requirement of teaching is to understand the subject content to be taught, and Goulding *et al.* (2003) in their research into trainee teachers' subject knowledge, conclude that strengthening the knowledge of trainees to support their teaching should be an aim of Initial Teacher Training Institutes. Few would argue with this, but it points out a difference between the mathematics knowledge that teachers bring to the classroom and the mathematics knowledge needed for teaching.

The small-scale research study of serving teachers, *'Making Connections: Effective Teaching of Numeracy'* (Askew *et al.* 1997), demonstrated that the teachers in classes that made the greatest gains had a rich understanding of the mathematics that they taught. They found no correlation between the teachers' level of mathematics qualification and their effectiveness as teachers, but they did find that there was a positive association between average class gains and the amount of CPD in which the teachers had engaged. The *Schools White Paper* (DfE 2010) agrees with the importance of the continuing development of teachers once they are in the classroom, but also asserts that the level of qualification is important, with a need to raise the minimum graduate entry to teaching. This is based on the findings of Allington and Johnson (2000), with research concluding that effective teachers need to have good prior academic attainment, combined with interpersonal skills, a willingness to learn and the motivation to teach. However, this goes against the weight of the research which asserts that prior academic attainment alone has little impact on the depth of subject knowledge of primary teachers, with deep subject knowledge developed while training to teach, when teaching and planning in the classroom, and through CPD (Aubrey 1994, Askew *et al.* 1997, Rowland *et al.* 2009, Ball *et al.* 2008).

How does the MaST Programme aim to develop participants deep subject knowledge?

Following an independent review of mathematics teaching in early years settings and primary schools, the final report from Sir Peter Williams recommended the training of maths specialist teachers in primary schools to '…articulate and share a clear vision for mathematics within the school' (Williams 2008: 20).

In response to these findings, the national two-year Masters level MaST Programme was introduced, involving Higher Education Institutions (HEI) around the country and linked Local Authorities to work with the participants in their schools. Teachers apply to join the programme and are selected from the strength of their application. From discussions I had with participants in the early meetings of the programme, the majority of the successful applicants were experienced teachers with a desire and enthusiasm to develop their own mathematics teaching and to have an impact on the mathematics teaching in their schools. Increasing subject knowledge was one of the aims of the programme:

> '… to develop a secure and deep knowledge and understanding of the mathematics that falls within the primary curriculum and extends into the EYFS and into KS3 and KS4' (Morgan 2010: 1).

The regional HEI providers involved with planning this programme took a connectionist approach to mathematics (Askew *et al.* 1997), focusing on five 'big ideas' that span and connect the mathematics curriculum. The aim was for teachers to develop deeper insight and have a clearer idea of the 'big picture' of mathematics. The five ideas, or key themes, are:

- Mathematical thinking
- Proportionality
- Pattern
- Generality
- Representation

These are all powerful aspects of mathematics, encompassing what it means to think mathematically, to problem-solve and to make sense of the mathematics curriculum. Making connections is key to developing a deep knowledge of these ideas. Learners may use different representations of multiplication, for example, using arrays, using number lines and by grouping items. However it will only make sense to the learners if they can use reasoning and mathematical thinking to demonstrate the connections between the representations. Learners may create a pattern of sticks and counters but it is the ability to generalise that moves the learning on and shows that they are thinking mathematically.

To link with the five big ideas, the programme also focuses on eight key pedagogies, which the course providers refer to, model, develop and apply within each module.
They are:

Prompting children's thinking through questions:
1. What do you notice?
2. What is the same and what's different?

Enabling learning through:
3. Drawing attention to…
4. Developing reasoning and making connections

Providing opportunities for children to:
5. Manipulate, experience, see (mathematics)
6. Engage in talk (listen, analyse and discuss)

Developing children's thinking through:
7. Investigation
8. Scaffolding

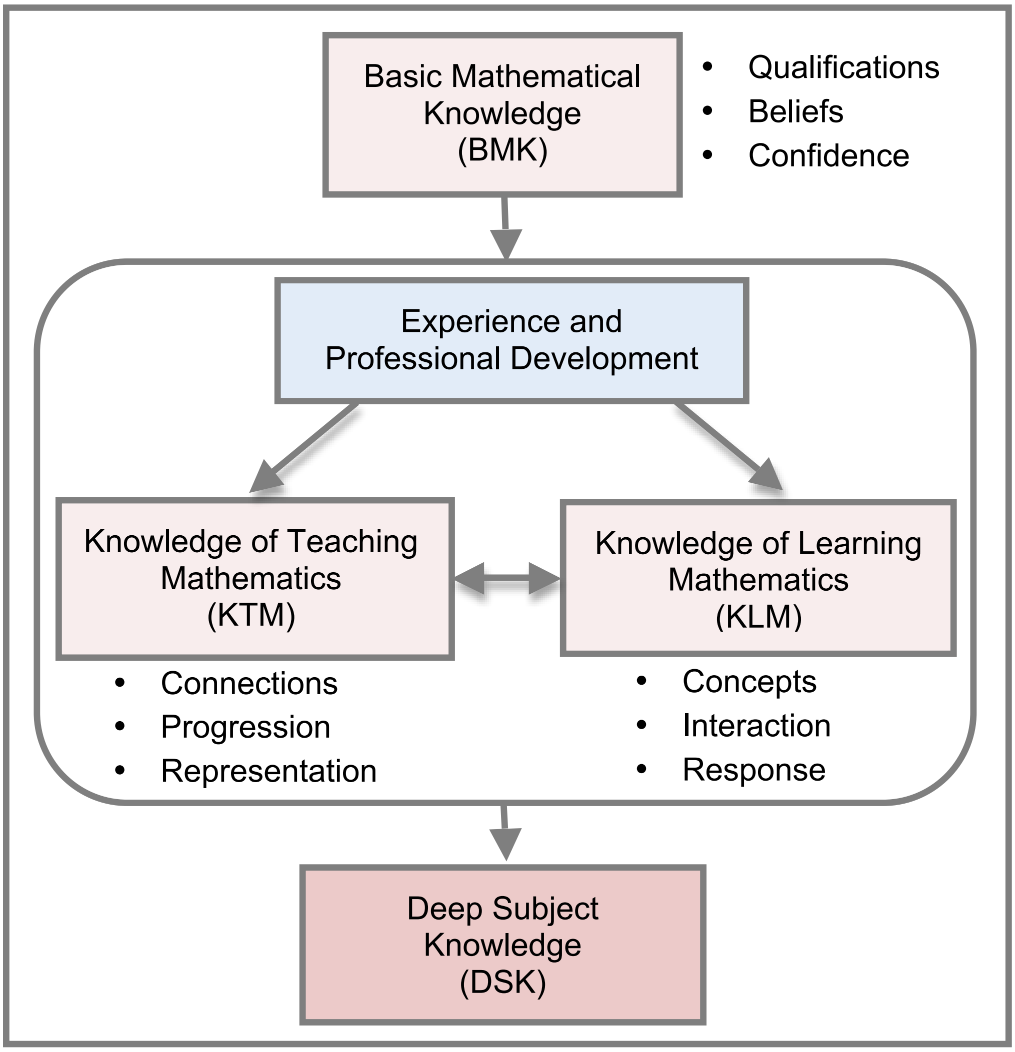
Referring to these as 'pedagogies' is possibly a little contrived and the grouping is confusing. Using the question 'What do you notice?' is an excellent and simple way to allow children to communicate, demonstrate their understanding and show their thinking. I am not sure it is a 'key pedagogy' however, and it is very different from the broad areas of investigation and scaffolding.

The MaST programme involves a variety of forms of professional development, including HEI weekend training sessions, LA network meetings, classroom observations and self-study. This includes a professional learning log to reflect and analyse their own learning and progress, including small-scale research, which is in line with the findings of Aubrey (1994) and De Geest (2011). The assignments are expected to be completed at Masters level, with specific learning outcomes and assessment criteria for the successful completion of each module. If successful at the end of the 2-year programme, participants will be awarded sixty credits towards the completion of a Masters degree.

Conclusion

From the research on the connections between subject knowledge and pedagogy, it is evident that deep subject knowledge should help a teacher become more effective in the classroom. The *Knowledge Quartet* (Rowland, Huckstep and Thwaites 2003) provides a useful model to support the evaluation of a teachers' level of subject knowledge but it is not explicitly structured to identify the nature of *deep* subject knowledge. Figure 2.3 illustrates my assertions of the process that teachers go through when developing their deep subject knowledge, with a particular emphasis on the impact of experience and CPD on their knowledge of teaching mathematics and on their knowledge of children's learning. The elements within each domain (BMK, KTM and KLM) provide the structure of an analytical framework for classroom observations and interviews to help identify the nature of deep subject knowledge.

Figure 2.3: Deep subject knowledge model



Each element of the model is supported by research evidence on subject knowledge detailed earlier in this literature review or through further reading:

1. Basic Mathematical Knowledge (BMK)

a. *Qualifications:* the level of mathematics qualification reached, including school and university mathematics qualifications (DfE 2010).

b. *Beliefs:* the attitudes and beliefs the teacher has towards mathematics, any negative emotions they have, their enthusiasm and whether they favour a discovery, transmission or connectionist approach (Aubrey 1994, Askew *et al.* 1997, Bibby 2002, Williams 2008).

c. *Confidence:* the mental skill, basic knowledge and general 'feel' for mathematics that a teacher has and brings to the classroom and whether they have a relational rather than an instrumental understanding of mathematical concepts (Skemp 1976).

2. Knowledge of Teaching Mathematics (KTM)

a. *Connections:* when planning and teaching a mathematics topic, an awareness of the possible mathematical connections and the cross-curricular links for each skill and concept (Shulman 1987, Ball 1990, Askew *et al.* 1997, Rowland *et al.* 2003).

b. *Progression:* knowledge of the scope and sequence of the curriculum, with an understanding of the mathematics being taught within the 'big picture' of the whole curriculum (Shulman 1987, Rowland *et al.* 2003).

c. *Representation:* the way the knowledge is explained or represented to the pupils to aid understanding, using language, images, action on objects and symbols (Shulman 1987, Rowland *et al.* 2003, Haylock 2005, Barmby *et al.* 2009, Alexander 2010).

3. Knowledge of Learning Mathematics (KLM)

a. *Concepts:* an understanding of mathematical concepts and skills, including dealing with misconceptions and individual needs and the ability to guide children in appropriate small steps forward in knowledge or understanding (Williams 2008, Ofsted 2008).

b. *Interaction:* the quality of discussion, pupil talk, reasoning and language, asking appropriate questions (Williams 2008, Alexander 2010).

c. *Response:* dealing with questions as they arise from pupils, flexibility, decision-making and the ability to 'think on your feet' or 'grab the moment' (Williams 2008, Ofsted 2009, Alexander 2010).